Mead

COMPOSITION

2451625.4 2451661.666

LOGBOOK # 62 27 APR 2000

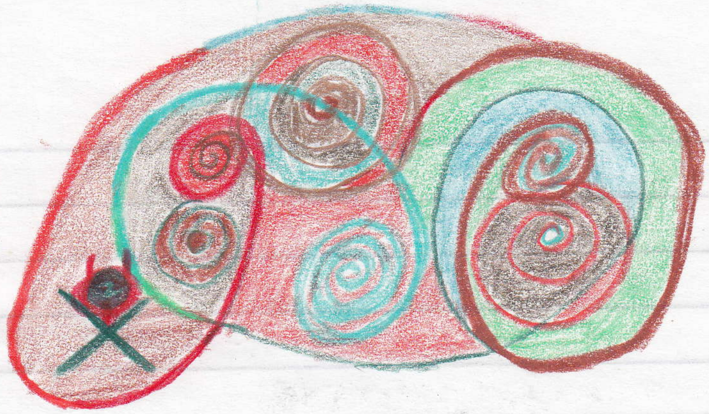
21 MAR 2000

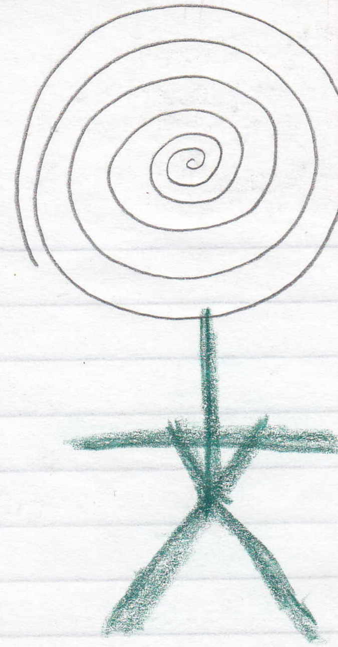
100 sheets • 200 pages
9¾x7½ in/24.7 x 19.0 cm
college ruled • 09932

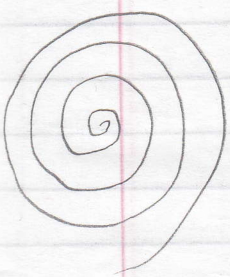
0 43100 09932 1
B-101

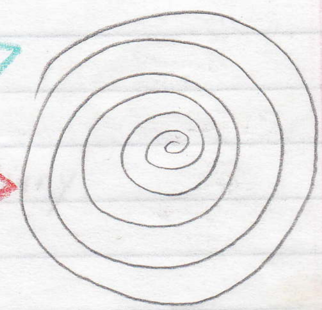
ORGANIC COMPUTER

COLLEGE RULED









i am born to die.

why?

i awaken

i wonder

i despair

i sleep

i die

confusion

honesty

complexity

honesty

organic computer

A Michael William Hentrich diary

L 62

ZONES

zone zero ∅

Nothing Need Be Done

2000.081.2
03.21

if $\det(A) = \emptyset$ matrix A is singular. if $\det(A) \neq 0$, A is invertible.
if A is invertible, then A is square.
properties of determinants:

① $\det(I) = 1$

② exchange rows → reverse the sign of the determinant
hence, the permutation of the identity matrix is either 1 or −1
depending on whether the number of ~~exch~~ permutations are
even or odd. odd # permutations → $\det(P) = -1$
even # permutations → $\det(P) = 1$

KEY ③ TWO PARTS

③a $\begin{vmatrix} ta & tb \\ c & d \end{vmatrix} = t \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

③b $\begin{vmatrix} a+a' & b+b' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}$

about linear combinations of the first row only, LINEAR FOR EACH ROW.

The determinant is a linear function.
Yet $\det(A+B) \neq \det(A) + \det(B)$
Only one row is different, the rest of the $n-1$ rows are same.

From these three properties of determinants, we can learn alot more about the NATURE of the function we call det().

④ 2 equal rows ⟶ $\det = \emptyset$
exchange rows, reverse signs. If $D = -D$, then $D = \emptyset$

property five is another key.

(5) subtract multiple ($l *$ row 1) from another row k.
I can get zeros below the pivots (diagonals) with these elimination steps, and the determinant does not change.

$$\begin{vmatrix} a & b \\ c-la & d-lb \end{vmatrix} = a(d-lb) - b(c-la) = ad - alb - bc + bla$$
$$= ad - bc$$

$$= \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a & b \\ -la & -lb \end{vmatrix}$$

$$= \begin{vmatrix} a & b \\ c & d \end{vmatrix} - l \begin{vmatrix} a & b \\ a & b \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} - \emptyset$$

(6) complete row of zeros $\longrightarrow$ $\det(A) = 0$
proof with 3a with ~~t=0~~ let $t = 5$

$$\begin{vmatrix} 0 & 0 \\ c & d \end{vmatrix} \xrightarrow{t * \text{row } 1} 5 \begin{vmatrix} 0 & 0 \\ c & d \end{vmatrix} = \emptyset$$

or let $t = 0$ in $\begin{vmatrix} ta & tb \\ c & d \end{vmatrix} \rightarrow t \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

(7) these $*$'s don't matter

$$\det U = \begin{vmatrix} d_1 & * & * & * \\ 0 & d_2 & * & * \\ 0 & 0 & \ddots & * \\ 0 & 0 & 0 & d_n \end{vmatrix} = \left(d_{11} \, d_{22} \cdots d_{nn} \right)$$

This is how MATLAB computes the determinant:
elimination $\rightarrow$ make it triangular $\rightarrow$ product of pivots.
All the off diagonals are killed off.

see how "7" works. Use 5a to do elimination to get U, then use property 3a to factor out the d's, and property 1 : $\det(I) = 1$ eliminate off diagonals

$$d_{11} d_{22} d_{33} \cdots d_{nn} \begin{vmatrix} 1 & & \\ & 1 & \\ & & \ddots \\ & & & 1 \end{vmatrix}$$

(8) $\det(A) = \emptyset \rightarrow$ matrix A is singular : row of zeros

$\det(A) \neq \emptyset \rightarrow$ matrix A is invertible.

It is invertible when echelon A produces full pivot rows

$\rightarrow U \rightarrow D \rightarrow d_{11} d_{22} \cdots d_{nn}$

Analyze : What are pivots of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$?

a is the first pivot. fine, what about the second pivot ?

$$\begin{matrix} a & b \\ c & d \end{matrix} \rightarrow \begin{matrix} a & b \\ 0 & d - \frac{bc}{a} \end{matrix} \longrightarrow ad - bc \;!!!$$

We finally are permitted to discover that the determinant of the 2×2 matrix is the product of $(a) * \left(d - \frac{bc}{a}\right)$, the product of the pivots !

Wow... we derived this by the properties themselves, not from some formula.

(9) $\det(AB) = \det(A)\det(B)$ hence $\det(A^{-1}A) = \det(I) = \det(A)\det(A^{-1})$

$1 = \det(A)\det(A^{-1})$ $\quad \det(A^{-1}) = 1 / \det(A)$

(10) $\det(A^T) = \det(A)$

original input. This is ONE-TO-ONE, 1-1, INJECTION.

$f: A \to B$ then f is injection (1-1) if and only if $f(x) = f(y)$ implies that $x = y$ for every $x, y \in A$.

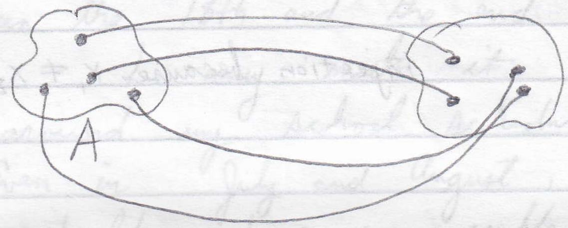
More consciely: $\left[f \text{ is 1-1 iff } f(x)=f(y) \rightarrow x=y \ \forall x,y \in A \right]$

contrapositive: Something is an injection iff $f(x) \neq f(y)$ whenever $x \neq y$.

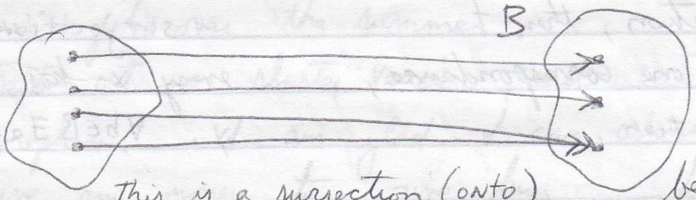
apriori assumptions $\quad x \neq y \longrightarrow f(x) \neq f(y)$

Defn $f: A \to B$ is a surjection (onto) if and only if $\forall b \in B \ \exists a \in A$ such that $f(a) = b$

codomain = range, $C(f) = R(f)$



A function is called a BIJECTION if it is both an injection and a surjection (both "one to one" and "on to".



This is a surjection (onto) because $\forall b \in B \ \exists a \in A$ such that $f(a) = b$

but it is not an injection (not 1-1) because there exists $f(a_3) = f(a_4)$ where $a_3 \neq a_4$.

BIJECTIONS: What does inverse imply?

If I have a function $f: A \to B$ for some $a \in A$ $f(a) = b$ then the inverse image of b should be the unique $a \in A$ such that $f^{-1}(b) = a$.

If f is a bijection, then f^{-1} exists.

To find inverse, switch x and y and solve for y again.

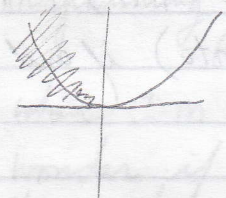
084.0210

$f(x) = x^2$ ($f: \mathbb{R} \to \mathbb{R}$) is not invertible

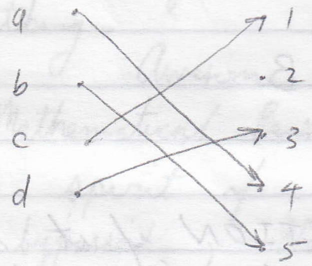
We can make it a function only by restricting the domain $D(f) = [\emptyset, \infty)$
$G(f) = [\emptyset, \infty)$

Injection and Surjection have to do with carefully defining the domain and codomain (image).

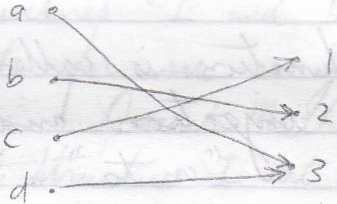
$f: [\emptyset, \infty) \to [\emptyset, \infty)$ IS invertible



$f^{-1}(x) = \sqrt{x}$



injection because $x_1 \neq x_2 \to f(x_1) \neq f(x_2)$



If both an injection and a surjection, then f is a one to one correspondance, or a bijection.

surjection because every x has a corresponding y. $\forall b \in B\ \exists a \in A$ s.t. $f(a) = b$

An injection passes both vertical and horizontal line tests.

Now, even though it is nearly 3AM and Dad will be calling at 5AM, I still want to try to start that M300 proof. If I can finish it, then I am free to work on M250 5.2 and LAB #5 Friday night. I will not be able to work on MATLAB Saturday night, so I will work on LAB #5 Friday night, then 5.2 problems in Freehold Saturday night. I am still under some kind of spell... that young woman ~~had~~ has gotten into my head.

18

Strang 19:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & 0 \\ c & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & d \end{vmatrix} = \underbrace{\begin{vmatrix} a & 0 \\ c & 0 \end{vmatrix}}_{\text{zero}} + \begin{vmatrix} a & 0 \\ 0 & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & 0 \end{vmatrix} + \underbrace{\begin{vmatrix} 0 & b \\ 0 & d \end{vmatrix}}_{\text{zero}}$$

$ad - bc$

We behold the METHOD (man). Now we can find the determinant formula for any $n \times n$ matrix!

$n!$ terms (non zero)

Split this up like crazy. A bunch of the $3^3 = 27$ pieces will be zero. When will we not have zero?

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

What are the survivors?

$$\begin{vmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{vmatrix} \text{ is a survivor.} = a_{11} a_{22} a_{33}$$

The survivors have one entry in each row and each column.

another survivor:

$$+ \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & 0 & a_{23} \\ 0 & a_{32} & 0 \end{vmatrix} = a_{11} a_{23} a_{32} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= (-1)\, a_{11} a_{23} a_{32}$$

Similar to a permutation matrix.

$$+ \begin{vmatrix} 0 & a_{12} & 0 \\ a_{21} & 0 & 0 \\ 0 & 0 & a_{33} \end{vmatrix} = (-1)\, a_{12} a_{21} a_{33}$$

$$+ \begin{vmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & 0 \\ a_{31} & 0 & 0 \end{vmatrix} = (-1)\, a_{31} a_{22} a_{13}$$

$$+ \begin{vmatrix} 0 & 0 & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & 0 \end{vmatrix} = (+1)\, a_{13} a_{21} a_{32}$$

$\det(A) = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31}$

$- a_{31}a_{22}a_{13} + a_{13}a_{21}a_{32}$, and hence, Gilbert Strang's "Big Formula" is: $\det(A) = \sum \pm a_{1\alpha}a_{2\beta}a_{3\gamma} + \dots + a_{n\omega}$

sum of n factorial terms

where do the n factorial terms come from?

Listen up. It is the nature/pattern of "$n!$".
There are $n!$ surviving terms because the chosen one from the first row can be chosen in n different ways. The chosen one from the second row can only be chosen in $n-1$ ways ... and so on.
The n column numbers are each used once

$(\alpha, \beta, \gamma, \dots, \omega) =$ PERMUTATIONS OF $(1, 2, 3, \dots, n)$

"choosing someone from every row and column amounts to permutations" where, for example,

$\begin{vmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{vmatrix}$ represents when $\alpha = 1$, $\beta = 2$, and $\gamma = 3$.

permutate $(4, 3, 2, 1) \rightarrow (1, 2, 3, 4)$ yields $+1$

$A = \begin{vmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix}$ $P(3, 2, 1, 4) \rightarrow -1$ 1 exchange

22 terms were zero
1 term was -1
1 term was $+1$
$\det(A) = \emptyset$

I still love mathematics, especially when I move slow.

Note: If I can view Strang #19 before leaving for Freehold, then I can work on 5.2 tonight and look over Calculus tomorrow. Tomorrow, when I return to Highland Park, I can finish that M300 proof and do Question 3 of lab 5.
By Tuesday evening I will have my VW back.

I will not bring my M300 text book. I want to focus on Linear Algebra and then meditate upon Lagrange Multiplier, but I will write the problem I am to prove "by Tuesday night". I will write it at the top of page 22 $\longrightarrow$

Strang preaches about Cofactors. This man is a true "professor of mathematics". He professes.

$$\det(A) = \underset{\text{FACTORS}}{a_{11}} \underset{\text{COFACTORS}}{\left(a_{22}a_{33} - a_{23}a_{32}\right)} \qquad \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{vmatrix}$$

$$\cancel{+} + a_{12}\left(-a_{21}a_{33} + a_{23}a_{31}\right) \qquad \begin{vmatrix} 0 & a_{12} & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & 0 & a_{33} \end{vmatrix}$$

$$+ a_{13}\left(a_{21}a_{32} - a_{22}a_{31}\right)$$

minus sign built into it

$$\begin{vmatrix} 0 & 0 & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 0 \end{vmatrix}$$

$$\text{Cofactor of } a_{ij} = C_{ij}$$

$$= \pm \det\begin{pmatrix} n-1 \text{ matrix} \\ \text{with row } i, \\ \text{column } j \text{ erased} \end{pmatrix} \qquad \begin{vmatrix} + & - & + & - & + \\ - & + & - & + & - \\ + & - & + & - & + \\ - & + & + & + & - \end{vmatrix}$$

$$\left.\begin{array}{l} + \quad \text{if} \quad i+j \text{ is even} \\ - \quad \text{if} \quad i+j \text{ is odd} \end{array}\right\} \quad (-1)^\wedge(i+j)$$

What is the "cofactor" in parentheses? called a "minor"? Strang doesn't care for the terminology.

Cofactor Formula : $\boxed{\det(A) = a_{11}C_{11} + a_{12}C_{12} + \dots + a_{1n}C_{1n}}$

8.2.

My stomache is feeling a little better. I was lucky to get a decent parking spot on the deck (CAC) next to Commons & Computer Lab. Last homeworks 6.3 Equivalence Relations worth 35 pts. The class did not do well, nor did I. 23 pts was highest grade; it was my grade: $23/35 = 66\%$. Big problem: do not use $x \sim y$ when more than one relation is involved.

Use xRy, xSy or $(x,y) \in R$ ~~and~~ $(x,y) \in S$,

Well, this is proving to be a most difficult semester, and yet as an individual I appear to be slightly above ... the bell curve is skewered.

Generally I feel fine. I am taking care of what has to be taken care of. I EXPECT these last 5 weeks to be brutal, I am bracing myself. This weekend I will devote my self to my studies, i.e, to Mathematics.

Note: I have 40 meals left on my card. $40/5 = 8$ meals per week!

Perhaps I will drive over to catch breakfast when I can. We'll see. Now I will smoke an American Spirit cigarette while reading Nietzsches <u>The AntiChrist</u> before coming in to view Strang_{20} — the last lecture on determinants. I will write up notes herein, and then begin 5.3 probs. I would like to complete Calc3 12.8 problems tomorrow.

Nietzsche: "We free spirits – we have that instinct and passion for INTEGRITY which makes war upon the "holy lie" even more than upon all other lies..."

Note: After Strang_{20}, before 5.3 problems, redo last quiz on scratch paper, then – after 5.3 – redo in tZ (last quiz)

Perhaps I will drive to the grocery store before midnight. Note: 5.3 Probs just up to 13.

Formula for A^{-1}

2×2 : $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\frac{1}{\det(A)}$

think cofactors

this is a matrix of cofactors

$d \to C_{11} \to \begin{bmatrix} a & b \\ c & d \end{bmatrix} \; d \; (-1)^2$

$-b \to C_{21} \to \begin{bmatrix} a & b \\ c & d \end{bmatrix} \; -b \; (-1)^3$

etc $-c \to C_{12} \to \begin{bmatrix} a & b \\ c & d \end{bmatrix} \; -c \; (-1)^3$

$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T = \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix}$

MAGIC FORMULA

$$A^{-1} = \frac{1}{\det(A)} C^T$$

$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1}$

Think:
What is cofactor of a?
$C_{11} = ei - fh$

The determinant of A involves products of n entries
The cofactor matrix involves products of $n-1$ entries.
Let us check the magic formula $A^{-1} = \frac{1}{\det(A)} C^T$
Check $AC^T = \det(A) I$

$$\begin{bmatrix} \boxed{a_{11} & \cdots & a_{1n}} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} C_{11} & \cdots & C_{n1} \\ C_{12} & & C_{n2} \\ C_{1n} & & C_{nn} \end{bmatrix} = \begin{bmatrix} \det(A) & & \emptyset \\ & \det A & \\ \emptyset & & \det(A) \end{bmatrix}$$

COFACTOR FORMULA $\det(A) = a_{11} C_{11} + a_{12} C_{12} + \dots + a_{1n} C_{1n}$

$\det(A) = a_{21} C_{21} + a_{22} C_{22} + \dots \; a_{2n} C_{2n}$

Why should it be that one row times the cofactors of a different row (which is now a column due to the transpose) be automatically zero? Why does that happen?

check it out:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} -b \\ a \end{bmatrix} = -ab + ba = \emptyset \,!$$

$$A_{\text{SCREWEDUP}} = \begin{bmatrix} a & b \\ a & b \end{bmatrix} \qquad \det(A_{\text{screwed up}}) = ab + b(-a)$$

The reason I get a zero in the upper triangle.
The inverse changes when the matrix changes - of course.

$$A\vec{x} = \vec{b}$$

CRAMER'S RULE

$$\vec{x} = A^{-1}\vec{b} = \frac{1}{\det(A)} C^T \vec{b}$$

"Cramer just had a good idea"

$$x_1 = \frac{\det(B_1)}{\det(A)} \qquad B_1 = \begin{bmatrix} b_1 & n-1 \\ | & \text{columns} \\ b_n & \text{of } A \end{bmatrix}$$

$$x_2 = \frac{\det(B_2)}{\det(A)}$$

$$x_j = \frac{\det(B_j)}{\det(A)}$$

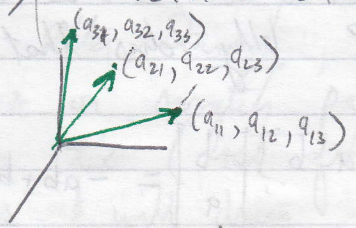
$$C_{11} \vec{b}_1 + C_{21} b_2 + \ldots$$

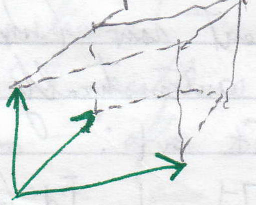
B_j = A with column j replaced by $\vec{b}$

Is Cramer's Rule useful in practice?
"Cramer's rule is a disastrous way to go!" - Strang
note: and yet Thursday's quiz will be on Cramer's rule.
The value is in A^{-1} and x.
Strang would use "elimination". He would never use Cramer's Rule. ok.

Next

3x3: $|\det(A)|$ = volume of a box ← proper word is parallelipiped





what does the sign tell us?

089. 0230 I am up to the last two problems in 5.3 Tomorrow, when I wake up and get some coffee in me, I will write up a couple "Cramers Rule" examples in t∫, and then I will begin 12.8 problems.

If I have time I will read about Multiple Integrals before the lecture. Now I am ready for another smoke, The Anti-Christ, and a peaceful slumber.

089.1330 I have been going over Cramer's Rule problems in t∫ #028. Now, before I get into Lagrange Multipliers, I want to show here that this is familiar:

Consider the general 2 by 2 linear system:

$$\left.\begin{aligned} a_{11}x + a_{12}y &= b_1 \\ a_{21}x + a_{22}y &= b_2 \end{aligned}\right\} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Intuitively we solve for x by eliminating y

$$\begin{aligned} (a_{22})(a_{11}x + a_{12}y = b_1) &\Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = a_{22}b_1 \\ (-a_{12})(a_{21}x + a_{22}y = b_2) & \quad -a_{12}a_{21}x - a_{12}a_{22}y = -a_{12}b_2 \end{aligned}$$

$$x(a_{11}a_{22} - a_{12}a_{21}) = a_{22}b_1 - a_{12}b_2$$

$$x = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

We solve in a similar manner for y. This is how I have solved linear systems since I can remember. here comes the connection... intellectual breakthrough

assuming that $a_{11}a_{22} - a_{12}a_{21} \neq 0$

$$x = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \begin{matrix} \leftarrow \det(B_1) \\ \\ \leftarrow \det A \end{matrix}$$

$$y = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \leftarrow \det(B_2)$$

Extending the pattern to a 3×3 linear system,

$$\underset{A}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}} \underset{\vec{x}}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{\vec{b}}{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}$$

$$x = \frac{\begin{vmatrix} \boxed{\begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix}} & \begin{matrix} a_{12} \\ a_{22} \\ a_{32} \end{matrix} & \begin{matrix} a_{13} \\ a_{23} \\ a_{33} \end{matrix} \end{vmatrix}}{\det A}, \quad y = \frac{\begin{vmatrix} \begin{matrix} a_{11} \\ a_{21} \\ a_{31} \end{matrix} & \boxed{\begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix}} & \begin{matrix} a_{13} \\ a_{23} \\ a_{33} \end{matrix} \end{vmatrix}}{\det A}$$

$$z = \frac{\begin{vmatrix} \begin{matrix} a_{11} \\ a_{21} \\ a_{31} \end{matrix} & \begin{matrix} a_{12} \\ a_{22} \\ a_{32} \end{matrix} & \boxed{\begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix}} \end{vmatrix}}{\det A} \qquad \text{generally } x_j = \frac{\det B_j}{\det A}$$

Another word for C^T, transpose of coefficent matrix, is "The CLASSICAL ADJOINT of a square matrix".

This is $C^T = [coef(a_{ij})]^T$

$$A^{-1} = \frac{C^T}{\det A}$$

I may return to this interesting topic later, after I return from Multiple Integrals lecture, but now I will take a little smoke break; then I want to work on Lagrange Multiplier problems.

089.1550 I have only 1 more problem to go on 12.8. After I finish, I will lie on my back and read section 13.1 (Multiple Integrals) to prepare for lecture. If I leave the house at 17:15, I can get coffee and park car by 17:45. I will eat chow after the lecture. When I return home I will write up M300 6.3 problems in TeX, and then review Cramer's Rule.

Over the weekend: M250 [Lab 5, 6.1] Thurs Night / Fri
M300 [Functions] Fri / Sat
M251 [13.1] Sat / Sun
Finish lab by Wed

1630 Even though I do not plan on minoring in mathematics, I am still glad to be taking both Multivariable Calculus and Mathematical Reasoning. I have always been curious...
The double integral of f over the rectangle R is

$$\iint_R f(x,y)\,dA = \lim_{\|P\|\to 0} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij}$$

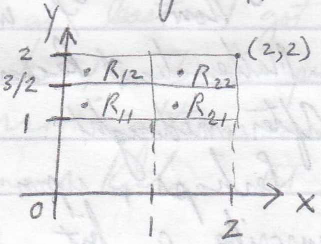
where $\|P\| = \max\{\Delta x_i\}$

$\Delta A_{ij} = \Delta x_i \Delta y_j$, hence $\iint_R f(x,y)\,dA = \iint_R f(x,y)\,dx\,dy$

example: Find an approximate value for the integral $\iint_R (x - 3y^2)\, dA$ where $R = \{(x,y) \mid 0 \le x \le 2,\ 1 \le y \le 2\}$ by computing the double Riemann Sum with partition lines $x = 1$ and $y = 3/2$ and taking (x_{ij}^*, y_{ij}^*) to be the center of each rectangle.

Note: I am doing this just to prepare for the lecture. If I can get over to the classroom across the river early, I will read The AntiChrist while I drink my coffee.

The partition:



The area of each subrectangle is $\Delta A_{ij} = \Delta x_i \Delta y_j = \frac{1}{2}$

(x_{ij}^*, y_{ij}^*) is the center of ~~each~~ R_{ij}

and $f(x,y) = x - 3y^2$

So the corresponding Riemann Sum is

$$\sum_{i=1}^{2}\sum_{j=1}^{2} f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij}$$

$$= f(x_{11}^*, y_{11}^*)\Delta A_{11} + f(x_{12}^*, y_{12}^*)\Delta A_{12} + f(x_{21}^*, y_{21}^*)\Delta A_{21} + f(x_{22}^*, y_{22}^*)\Delta A_{22}$$

$\frac{6}{4}$ $5/4$ $\frac{4}{4}$

$$= f\left(\tfrac{1}{2}, \tfrac{5}{4}\right)\Delta A_{11} + f\left(\tfrac{1}{2}, \tfrac{7}{4}\right)\Delta A_{12} + f\left(\tfrac{3}{2}, \tfrac{5}{4}\right)\Delta A_{21} + f\left(\tfrac{3}{2}, \tfrac{7}{4}\right)\Delta A_{22}$$

$$= \left(\tfrac{1}{2} - 3\left(\tfrac{25}{16}\right)\right)\tfrac{1}{2} + \left(\tfrac{1}{2} - 3\left(\tfrac{49}{16}\right)\right)\tfrac{1}{2} + \left(\tfrac{3}{2} - 3\left(\tfrac{25}{16}\right)\right)\tfrac{1}{2} + \left(\tfrac{3}{2} - 3\left(\tfrac{49}{16}\right)\right)\tfrac{1}{2}$$

$$= \left(\tfrac{8}{16} - \tfrac{75}{16}\right)\tfrac{1}{2} + \left(\tfrac{8}{16} - \tfrac{147}{16}\right)\tfrac{1}{2} + \left(\tfrac{24}{16} - \tfrac{75}{16}\right)\tfrac{1}{2} + \left(\tfrac{24}{16} - \tfrac{147}{16}\right)\tfrac{1}{2}$$

$$= -\tfrac{67}{32} + \left(-\tfrac{139}{32}\right) + \left(-\tfrac{51}{32}\right) + \left(-\tfrac{123}{32}\right) = -\tfrac{380}{32} = \tfrac{-95}{8}$$

$$\begin{array}{r} 95 \\ 4\overline{)-380} \\ 36 \\ 20 \end{array}$$

thus $\iint_R (x - 3y^2)\, dA \approx -11.875$

We shall see if Dr Norman Levitt can help us clarify this.

36

089. 2100 I am into all this math. As difficult, dry, and challenging as the material is, I am still enjoying being utterly consumed by it. This is a wonderful semester. I am not likely to forget it.

Some very awesome semesters lie in store for this organic computer. Over the summer, the extremely challenging Discrete Structures II ... It will put me in touch with concepts studied last summer. I will prepare for it in advance from May 15 to June 25.

2200 Question 3 of Lab #5 completed. Now I will write up Equivalence Relations problems from Mathematical Reasoning lecture in Techgnostic Scribblings. After midnight I will review Cramer's Rule. Perhaps, upon beginning Zone 2: Dark City, I will transcribe a bit of the multiple integral lecture notes I took tonight.

For me, it was an incredibly special moment. I had to hold back my excitement. I am really, really into it.

$$\text{volume} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum_i f(x_i, y_i)\,\Delta x_i\,\Delta y_i = \iint_R f(x,y)\,dA$$

The process will converge to a limit.

We will assume the region is bounded so as to keep the integral (a number) from blowing up into infinity.

$$\iint f(x,y)\,dA = \text{volume} = \int_a^b A(x)\,dx$$

$A(x_0) = \int_c^d f(x_0, y)\,dy$ where x_0 is treated as a constant.

$$\text{volume} = \int_a^b \left[\int_c^d f(x,y)\,dy\right] dx \Rightarrow \text{an honest to god number}$$

Customary to omit brackets: $\int_a^b \int_c^d f(x,y)\,dy\,dx$

092. 2045 After a shower I feel better. If I can get all the M251 13.2 and 13.3 problems completed tonight I will be happy. I can then tighten up on M250 5.3 problems, finish MATLAB, and try to make some sense of M300 problems (all this tomorrow of course).

Now that my brain has been attacking this scheduling classes problem for the past 2 days, now that my brain has come up with a solution, I can be glad I am getting the second grant. I am very much looking forward to Summer 2000, Fall 2000, Spring 2001, Summer 2001, Fall 2001, and especially Spring 2002. I am also prepared to go to school at night in the summer of 2002 should I have to.

Now, Multivariable Calculus: Multiple Integration. This course I do not need for my major, but I have taken it solely to give me room to choose numerical analysis concentration should I be drawn to it. I will do the work in my sketch diary, but check out the first problem:

Calculate the iterated integral: $\int_{-1}^{1}\int_{0}^{1}(x^3y^3 + 3xy^2)\,dy\,dx$

I see that $A(x_0) = \int_0^1 (x_0^3y^3 + 3x_0y^2)\,dy = \left[x^3\frac{y^4}{4} + 3x\frac{y^3}{3}\right]_{y=0}^{y=1}$

$A(x_0) = \frac{x^3}{4} + x =$ ~~[illegible]~~

hence $\int_{-1}^{1}\int_0^1 (x^3y^3+3xy^2)\,dy\,dx = \int_{-1}^{1}\left(\frac{x^3}{4} + x\right)dx$

$$= \frac{1}{4}\frac{x^4}{4} + \frac{x^2}{2}\Big|_{x=-1}^{x=1} = \frac{x^4}{16} + \frac{8x^2}{16}\Big|_{x=-1}^{x=1} = \left(\frac{1+8}{16} - \frac{1+8}{16}\right) = 0$$

Not too bad. I have a "feel" for this. I will do the work in pencil in my M251 sketch diary. The Tide is Turning. After a few of these problems, I will make a Dunkin Donuts run. I ~~am~~ am very happy about the second grant. The tide is definitely turning Mr Waters. I believe the "well-rounded" education will stimulate me. I will reap the rewards won by hard experience.

"The object now is to reap as ~~rich~~ rich and complete a harvest as possible from the days of experiment and hard experience." — Friedrich Nietzsche.

It feels good to me...

I can do nothing to erase those regretful experiments. Only the passage of time can put things in the proper perspective. The experiences I have endured have been hard, and that is why I embrace the challenges before me. I am up to the task!

Although I am EXCITED about the coming semesters, I will, at most, keep only 2 terms in VIEW. Those are the summer 2000 and fall 2000 terms. Imagine how intense this summer will be! And I will be working with my Dad during the day — but focusing ALL MY MENTAL POWER on Discrete Structures II.

There is no doubt that I will be fascinated by Numerical Analysis & Computing, but that semester will be so full of diversity as well. Imagine that, come September I will be some revolutionary thinker...

fiction & ideology ...

and human evolution is a cool course as well.

Look at me, so enthusiastic! It is time to work on Multiple Integrals.

092.2300 I have to pause here from my work just to make a note of something I find very beautiful. I find beauty in these math problems. Also, I stare at page -1 of this L62, the list, and I am filled with joy and zest for life. I am glad I will have a well rounded education.

THIS ORGANISM MAY BRING SOME LOVE FOR LEARNING TO Rutgers

Observe this problem: $\int_0^{\ln 2}\int_0^{\ln 5} e^{2x-y}\,dx\,dy$

$$= \int_0^{\ln 2}\left(\tfrac{1}{2}e^{2x-y}\Big|_{x=0}^{x=\ln 5}\right)dy = \int_0^{\ln 2}\left(\tfrac{1}{2}e^{2\ln 5-y} - \tfrac{1}{2}e^{-y}\right)dy$$

now, not only does $e^{\ln 5} = 5$, but $e^{2\ln 5} = 25$, and we can split $e^{2\ln 5 - y}$ into $25e^{-y}$!!!

so, $\int_0^{\ln 2}\frac{1}{2}\left(25e^{-y} - e^{-y}\right)dy = \int_0^{\ln 2} 12e^{-y}\,dy$

so beautiful...

now, $\int_0^{\ln 2} 12e^{-y}\,dy = -12e^{-y}\Big|_{y=0}^{y=\ln 2}$

(because $\int e^u\,du = e^u$ and $\int e^{kx}\,dx = \frac{1}{k}e^{x}$)

$$= -12\left(e^{-\ln 2} - e^{0}\right) = -12\left(\tfrac{1}{2} - 1\right)$$

$$= -12\left(-\tfrac{1}{2}\right) = 6$$

and that, my friend, is beautiful!
Surely it has been worth the entire journey just to have made this little breakthrough. Such zest for learning I have. I may end up not needing Multivariable Calculus after all is said and done, but I am glad I have been exposed to it.

The same can be said for Mathematical Reasoning even though I am not enjoying it very much. I am glad that both Descrete Structures II and Numerical Analysis will be MATH-ORIENTED. I can't help but write about it. I am very into it. I am sure to be totally consumed by all 3 current math courses until May 15. This has been "the Term of the 640's"...

$\boxed{\begin{matrix}2000.093.0\\04.02.0030\end{matrix}}$ Note: at 2AM or so, Daylight Savings time begins. Fall back, spring ahead. So, at 0200 it will be 0300. Anyway, I thought this problem was cool in that it exposes some more characteristics of eulers constant and "exponents on exponents":

$$\iint_D e^{x/y}\, dA, \quad D = \{(x,y) \mid 1 \le y \le 2,\ y \le x \le y^3\}$$

$$\int_1^2 \int_y^{y^3} e^{x/y}\, dx\, dy = \int_1^2 y e^{x/y} \Big|_{x=y}^{x=y^3} dy$$

note the rule $\int e^{kx} dx = \frac{1}{k} e^{kx}$

well, $\int e^{\frac{1}{y}x} dx = \frac{1}{\frac{1}{y}} e^{x/y} = y e^{x/y}$

$$* \rightarrow \int_1^2 \left(y e^{y^2} - y e \right) dy = \left(\frac{1}{2} e^{y^2} - e \frac{y^2}{2} \right)_{y=1}^{y=2}$$

$$= \frac{1}{2} \left(e^{y^2} - e y^2 \right) \Big|_{y=1}^{y=2} = \frac{1}{2}(e^4 - 4e) - \frac{1}{2}(e - e)$$

$$= \frac{1}{2}(e^4 - 4e).$$

Now, the tricky part is $*$.

first of all, how is $\int y e^{y^2}$ equal to $\frac{1}{2} e^{y^2}$

This is the tricky part. It is clear that $\int ye = e\frac{y^2}{2}$

That was no problem.

but the form $\int x b^{x^2} dx$ is tricky.

If it were not for the coeffecent x, man!

But that coeffecent helps.

For $\int x b^{x^5} dx$, derive gives $\int x b^{x^5} dx$!!

For $\int x b^{x^2} dx$, derive gives $b^{x^2} / 2 \ln b$, and this was my clue. For $\frac{e^{y^2}}{2 \ln e} = \frac{e^{y^2}}{2}$ because $\ln e = 1$.

It is like $\log_{10} 10$.

I will investigate it further some other time.

093.0330 I finished M251 13.2 + 13.3. Tomorrow I can begin Question 4, MATLAB... I may even pull an "all nighter" and peck at it in the twilight zone.

One of the calculus problems involved integration by parts. Recall $\int u\,dv = uv - \int v\,du$, where $u = f(x)$, $du = f'(x)dx$, $v = g(x)$, $dv = g'(x)\,dx$. The formula does not need to be memorized as it is constructed directly from the product rule for differentiation.

recall that $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

In the notation for indefinite integrals, this becomes
$\int [f'(x)g(x) + f(x)g'(x)]\,dx = f(x)g(x)$

or $\int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx = f(x)g(x)$

We can rearrange this equation as

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

$$\int u\,dv = u\,v - \int v\,du$$

The problem was $\iint_D x\cos y\,dA$, where D is bounded by $y=0$, $y=x^2$, $x=1$. Here $x=\sqrt{y}$
$y=1$ so $0 \le y \le 1$ and $\sqrt{y} \le x \le 1$

so $\int_0^1 \int_{\sqrt{y}}^1 x\cos y\,dx\,dy = \int_0^1 \left(\frac{x^2}{2}\cos y\right)\Big|_{x=\sqrt{y}}^{x=1} dy$

$= \int_0^1 \left(\frac{1}{2}\cos y - \frac{y}{2}\cos y\right)dy = \int_0^1 \frac{1-y}{2}\cos y\,dy$

and at this point I needed Integration By Parts
let $u = \frac{1-y}{2}$ let $dv = \cos y\,dy \longrightarrow du = -\frac{1}{2}dy$, $v = \sin y$

Apply the formula: $\left(\frac{1-y}{2}\right)\sin(y) - \int \sin(y)\left(-\frac{1}{2}\right)dy$

$= \left[\frac{\sin(y) - y\sin(y)}{2} - \left(\left(\frac{1}{2}\cos(y)\right)\right)\right] = \left(\frac{\sin(y) - y\sin(y) - \cos(y)}{2}\right)\Bigg|_{y=0}^{y=1}$

093.0630 I guess I will be going to sleep soon. The sun is due to rise any minute now, and I just completed lab #5, I am printing now as the first couple of birds start their ancient ritual... I will sleep all day when I finally do sleep. Now, or later I should say - for I am truly done for "tonight" -, all I need to look at Sunday evening is M300 problems and a couple from M251 5.3 on volume. My old brain pulled through for me once again.

093.1600 - slept from 0700 to 14:30 - almost 8 hours. The mathematical reasoning (M300) work is such that I can not just jump right into it. I really have to think about it. Perhaps I will go over my notes from class while browsing the world wide web for information.

I will use this logbook to keep track of the work in progress. See p 47 for a second.

I understand that $A \subseteq f^{-1}(C)$ iff $f(A) \subseteq C$

If $A, B \subseteq X$, $f(A \cap B) \subseteq f(A) \cap f(B)$

if $C \subseteq D$ in Y then $f^{-1}(C) \subseteq f^{-1}(D)$

if $A \subseteq B$ in X $f(A) \subseteq f(B)$

Here we were asked to show that $f(A \cap B) = f(A) \cap f(B)$ iff f is injective for all $A, B \subseteq X$.

Some insight first: if $f: X \rightarrow Y$ is 1-1 and onto then the correspondence that goes backwards from Y to X is also a function and is called an inverse, denoted f^{-1}.

For $f: X \to Y$ 1-1 and onto we always have:

$\forall x \in X \ (f^{-1} \circ f(x) = x)$ and $\forall x \in Y \ (f \circ f^{-1}(x) = x)$

If $x \in Y$ looks strange at first, (remember that) we normally use $x \in X$

and $y \in Y$ only to emphasize the fact that we had two elements that came from two possibly different sets. In general, we can call elements of a set by any name we choose. Here we have two separate equations, so it's alright to use $x \in X$ in one of them, and $x \in Y$ in the other.

How would one verify these properties?

One web sit just gives the following:

Let $f^{-1} \circ f(x) = z$

We would like to show that $z = x$

Since $f^{-1} \circ f(x) = f^{-1}(f(x)) = z$ is equivalent to $f(x) = f(z)$ by the definition of inverse.

The injection, 1-1, property says that $z = x$.

a note about the composition of functions:

if $f: A \to B$ and $g: B \to C$

we get $g \circ f: A \to C$ and $(g \circ f)(a) = g(f(a))$

example: $f(x) = x^2$, $g(x) = \sin x$

$(g \circ f) = g(f(x)) = g(x^2) = \sin(x^2)$

If we represent f as a relation $R \subseteq A \times B$ and g by $S \subseteq B \times C$ then $g \circ f$ is represented by $S \circ R \subseteq A \times C$ since

$S \circ R = \{(a, c) \mid \exists b \in B$ with $(a, b) \in R$ and $(b, c) \in S\}$

$b = f(a)$

$c = g(b)$

$c = (g \circ f)(a) = g(f(a)) = g(b)$

093. 1700 Some very common sense termonology from Wei-Chi Yang : If a function passes the horizontal line test, then such a function is a one-to-one function, an injection. If a function is 1-1, such a function will have an inverse.

I may have to look through the library books I borrowed from the Math Library, those Discrete Mathematics for Computer Science books. It is hard to believe that in just 4 weeks, all these math lectures will be over.

Math does not go away though, just because there may not be any more MATH COURSES (640's), does not imply that there will be no more math. The 198's (COMP SCI COURSES) are quite theoretical and based in mathematics.

093.1815 I will return the books tomorrow as I will be at Busch Campus anyway. I think I will take G from D/C to Busch, drop the books off, go to UMDNJ for my appointment, then catch A or H back to CAC for some chow before walking home to exchange books. (M250 Texts for M251 texts). It will be another manic monday.

The entire semester has basically been a mental exercise, a "getting my brain" sharper — an upgrade for my mental processes. I wonder if I will be permitted to register for Numerical Analysis and Computing. Linear Algebra is a prerequisite and I haven't completed it yet. Perhaps I will have to go in Person on Monday, the 10th, right after my Linear Algebra exam. Note: around 9 or 10PM, go over 5.3 problems, and then relax so as to sleep by 1AM.

093.1920 No luck with the books. I am going to have to work it out here. I can't imagine what the exams are going to be like!

OK: Show that $f(A \cap B) = f(A) \cap f(B)$ iff f is injective for all $A, B \subseteq X$.

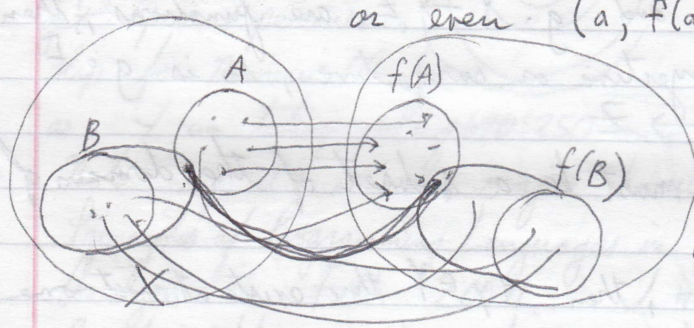
First of all, if f is injective $\forall A, B \subseteq X$, there is at most one $a \in A$ for each $f(a) \in f(A)$. Likewise the exists at most one $b \in B$ for each $f(b) \in f(B)$

$A \cap B \subseteq A$ and $A \cap B \subseteq B$

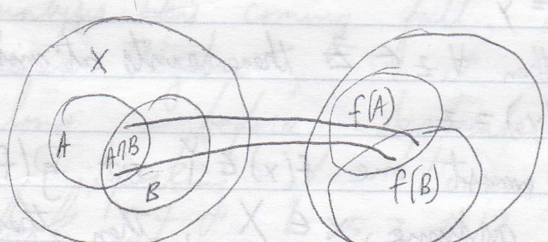
Generally, if $A, B \subseteq X$, then $f(A \cap B) \subseteq f(A) \cap f(B)$

$A \cap B$ corresponds to $(x, y) \in A \wedge (x, y) \in B$?

or even $(a, f(a)) \wedge (b, f(b))$?



Visually, here it is in a nutshell. But how to write a proof?



~~All ... f(B)~~

f is 1-1 so

$f(x) = f(y) \rightarrow x = y \;\forall x, y \in A$

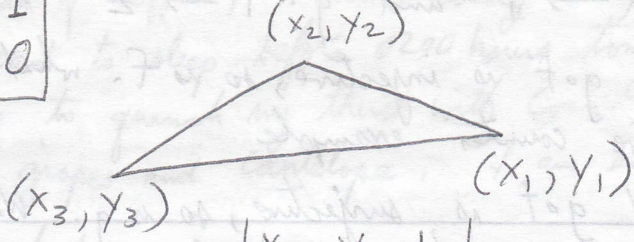
hence, when $f(a) = f(b)$, this is in $f(A) \cap f(B)$ so $a = b$ is in $A \cap B$. ☒

093.2030 Two down, one to go — a little depressed... Rather than coffee and pizza, why not cereal, hot cocoa? 2100, I will go for a drive to the foodstore just to take a little break from "my work", and then I will complete the third and final M300 problem. By 11PM I should be able to look over Linear Algebra.

Note: MAY WANT TO CALL ABOUT THAT { 1. M250 PREREQ C323
2. ADMITTANCE UCNB
3. FIN: TAG grant

64

2000.094.1
04.03.0000



area of triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

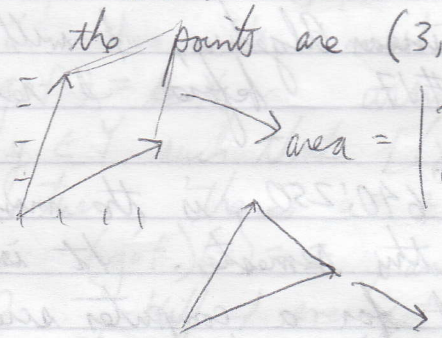
$= \frac{1}{2}(x_2y_3 - y_2x_3) - \frac{1}{2}(x_1y_3 - y_1x_3) + \frac{1}{2}(x_1y_2 - y_1x_2)$

which is written in the text as $+\frac{1}{2}(x_1y_2 - x_2y_1)$
$+\frac{1}{2}(x_2y_3 - x_3y_2)$
$+\frac{1}{2}(x_3y_1 - x_1y_3)$

I arrived at the first equation using
and $(-1)^{i+j}\,C$

Memorizing is ok, but intuitive feel is better.

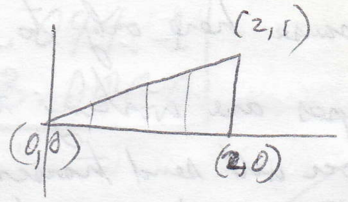
note that if the sides are given as vectors
$\vec{v} = (3,2)$ and $\vec{w} = (1,4)$
then the points are $(3,2)$ and $(1,4)$



area $= \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$

area $= \frac{1}{2}\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 5$

094.0100 I am not sleepy yet, but enough is enough. I have many phone calls to make (Don't forget Kathy Lambertson of DVR!) Now I will try to ~~and~~ unwind. How? SMOKE.



$$f(x,y) = xy$$

$$\iint_R xy \, dA$$

describe the region: $0 \leq x \leq 2$

$y = 1$ when $x = 2$
$y = 0$ when $x = 2$?

why does $0 \leq y \leq \frac{x}{2}$?

$$\int_0^2 \int_0^{x/2} xy \, dy \, dx = \int_0^2 \left[x \frac{y^2}{2} \right]_{y=0}^{y=x/2} dx$$

$$= \int_0^2 \left(x \frac{\left(\frac{x}{2}\right)^2}{2} \right) dx = \int_0^2 \frac{x^3}{8} dx$$

$$= \frac{x^4}{32} \Bigg|_{x=0}^{x=2} = \frac{16}{32} = \frac{1}{2}$$

Yes, but I am confused about the region $0 \leq y \leq x/2$.
Perhaps Chris Long will clarify.
Now, I have to run.
Note: Tonight work on M300 and M250 (6.1)

094. 2100 The problem was nothing like the above. Finding the bounds is the most difficult part.
~~*~~ YW_1 touched my arm in class. She is hot. YW_3 was very friendly tonight, and looking like a woman I could get close to. YW_2 came in and sat behind me. I was surrounded by the 3 of them.

83

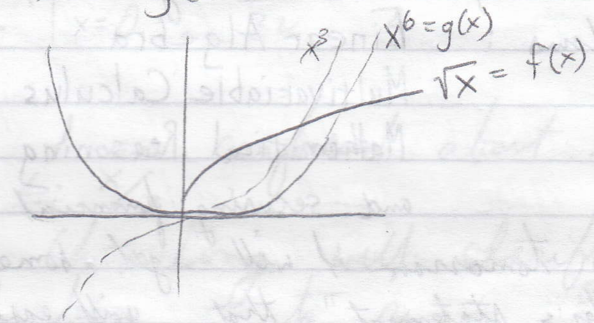
$\boxed{\begin{array}{l}2000.095.2\\04.04.0100\end{array}}$ I will be so relieved when I know for sure I will be getting financial aid. I will also be relieved when this semester is over. It has been very challenging to say the least. But I can't stop or ease up now. I have to suffer now rather than regret later for not putting me nose to the grindstone while I had the chance.

I have a tremendous amount of work before me, but it is only for 5 more weeks; after that I can zone out and study "creatively" for a full 7 weeks until Descrete Structures II — which I will be able to focus on 100%. For now, there is so much math in my brain, being sorted out internally.

0145 What about $f(x) = \sqrt{x}$

$$g(x) = x^6$$

then $(g \circ f)(x) = g(f(x)) = (\sqrt{x})^6 = x^3$



$g \circ f$ is bijective ~~g(x) is surjecti~~

and, hence ~~su~~ injective $x_1 \neq x_2 \longrightarrow f(x_1) \neq f(x_2)$

so is f, but $g(x)$ is not

This works for $g \circ f$ is surjective too

$g(x)$ is surjective: for all b, $\exists a$ s.t. $f(a) = b$

Not so for $f(x) = \sqrt{x}$

095-0200 I don't think I will be able to rest until I finish this problem. Tomorrow I will work on M250 6.1; but tonight I have to come up with some counter examples.

If $g \circ f$ is injective, so is f; but not g.

suppose $f : \{0, 1, 2\} \longrightarrow \{a, b, c, d\}$
$f(0) = a \quad f(1) = c \quad f(2) = d$

and $g : \{a, b, c, d\} \longrightarrow \{e, f, g, h\}$
$g(a) = e \quad g(b) = e \quad g(c) = f \quad g(d) = h$

then $(g \circ f) : \{0, 1, 2\} \longrightarrow \{e, f, g, h\}$

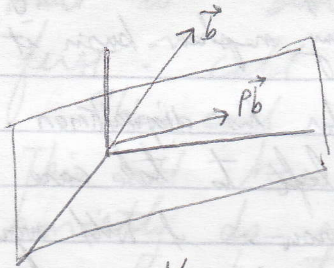
$(g \circ f)(0) = g(f(0)) = g(a) = e$
$(g \circ f)(1) = g(f(1)) = g(c) = f$
$(g \circ f)(2) = g(f(2)) = g(d) = h$

$1 \neq 2 \rightarrow f$??? ~~Not surjective~~ ~~Not injective~~

no, surjective means at least one x s.t. $f(x) = y$
injective means at most one x s.t. $f(x) = y$
Above, both 1 and 2 yield f.
But $g \circ f$ is not injective. Make it so and see what happens. Change $g(d) = f$ to $g(d) = h$.
Now $g \circ f$ is injective.
Shall I use this? YES. Write it up.

If $g \circ f$ is surjective, so is g. What about f?
$f : \{\lambda, \mu, \cancel{\theta}\} \longrightarrow \{\alpha, \beta, \gamma\}$
$f(\lambda) = \alpha, \quad f(\mu) = \beta$
$g : \{\alpha, \beta, \gamma\} \longrightarrow \{\pi, \omega\}$
$g(\alpha) = \pi, \quad g(\beta) = \omega, \quad g(\gamma) = \omega$
$(g \circ f) : \{\lambda, \mu\} \longrightarrow \{\pi, \omega\}$ — is never hit. Fix it.
$(g \circ f)(\lambda) = g(\alpha) = \pi \quad (g \circ f)(\mu) = g(\beta) = \omega$

We no longer have $A\vec{x}=\vec{b}$ because we have two unknowns now. We cannot use elimination. Let's look at a projection matrix:

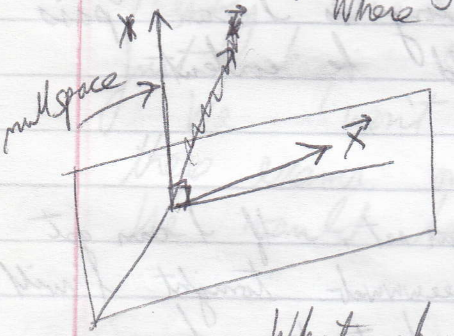


What are the x's and λ's for projection matrix?

We know that P projects us down to $P\vec{b}=\vec{p}$. Is $\vec{b}$ an eigenvector? NO

If we project onto the SAME PLANE, then we will end up with an eigenvector!

Any x in plane: $P\vec{x}=\vec{x}$ where $\vec{x}$ is an eigenvector, and the eigenvalue is 1. We have an entire plane of eigenvectors.



Where is there an eigenvector NOT in the plane?

Any $\vec{x} \perp$ plane $P\vec{x}=0\vec{x}$

The eigenvalues for the projection matrices are $\lambda_1=1$

$\lambda_2=0$

What about the permutation matrix?

$$A=\begin{bmatrix}0&1\\1&0\end{bmatrix}$$

What can we multiply by and ~~#~~ end up in the same direction?

A permutes x_1 and x_2, exchanges rows 1 and 2.

$$\vec{x}=\begin{bmatrix}1\\1\end{bmatrix} \rightarrow A\vec{x}=\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix} \rightarrow \lambda=1 \rightarrow A\vec{x}=\vec{x}$$

$$\vec{x}=\begin{bmatrix}-1\\1\end{bmatrix} \rightarrow A\vec{x}=\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}1\\-1\end{bmatrix} \rightarrow \lambda=-1 \rightarrow A\vec{x}=-\vec{x}$$

Fact: the sum of the n λ's $= a_{11}+a_{22}+a_{33}+\dots+a_{nn}$

In the 2 by 2 case, the "trace" tells us the other eigenvalue as soon as we know one.

How to solve $A\vec{x} = \lambda\vec{x}$. Simple idea: rewrite as $(A-\lambda I)\vec{x} = \vec{0}$

We don't know λ and we don't know $\vec{x}$, but we do know something.

We know that, if there is a nonzero $\vec{x}$, ~~so~~ that the A shifted by λI must be singular. Singular implies that the equations are multiples of each other. So $(A-\lambda I)$ is singular. We know that the determinant of singular matrices is zero.

Observe: $\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12-12=0$; $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{vmatrix} = 6-6+(-1)(6-6)+6-6=0$

Therefore, what we KNOW is that $\det(A-\lambda I)=0$

We have removed $\vec{x}$ from the equation.

This is the key equation, called the characteristic equation.

The idea is to find n lamdas first.

(there could be "repeated λ's". They are a source of trouble).

Once we find the $\vec{x}$'s once we have found the λ's?

We find the NULLSPACE. We do elimination, we identify the pivot values and the free variables...

$(A-\lambda I)$
SINGULAR

λI SHIFTS A

example: $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ a symmetric matrix will have real eigenvalues. The vectors are perpendicular.

In advance, we can see something... The diagonal is shifted by λ.

$\det(A-\lambda I) = \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 - 1 = 9-6\lambda+\lambda^2-1 = \lambda^2-6\lambda+8=0$

$(\lambda-2)(\lambda-4)=0$

First, though, what is this number 6 that is the coefficient of λ?

Is it not trace(A), the sum of the diagonals?

The sum of the diagonals will equal the sum of the eigenvalues! What is the number 8?

What is that constant term? It is the determinant of A, $\det(A) = 9-1=8$! $\lambda_1 = 4$, $\lambda_2 = 2$ (Note $2*4=8$)

Now we find the eigenvectors. They are in the nullspace

If we shift A by 4 or by 2, we get a singular matrix.

$(A-4I) = \begin{bmatrix}3&1\\1&3\end{bmatrix} - \begin{bmatrix}4&0\\0&4\end{bmatrix} = \begin{bmatrix}-1&1\\1&-1\end{bmatrix}$ a singular matrix

So what's the $\vec{x}$? The $\vec{x}$ is in the NULLSPACE! dimension $n-r$: $A\vec{x}=\vec{0}$.

$\begin{bmatrix}-1&1\\1&-1\end{bmatrix} A \begin{bmatrix}x_1\\x_2\end{bmatrix} = \vec{0} \longrightarrow \begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$

We are only choosing one vector in the nullspace of $(A-\lambda I)\vec{x}$ to form a basis of our eigenvectors.

$(A-2I) = \begin{bmatrix}3&1\\1&3\end{bmatrix} - \begin{bmatrix}2&0\\0&2\end{bmatrix} = \begin{bmatrix}1&1\\1&1\end{bmatrix}$

$\begin{bmatrix}1&1\\1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \longrightarrow \begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-1\\1\end{bmatrix}$

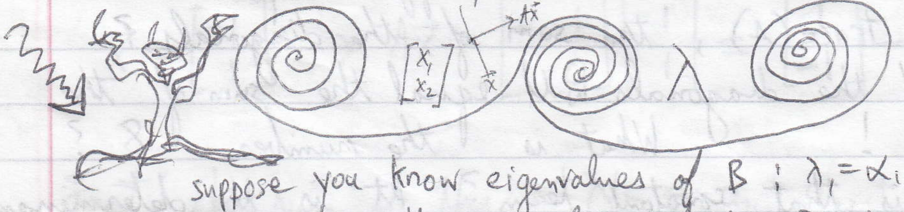
check it: if we did elimination on $\begin{bmatrix}1&1\\1&1\end{bmatrix}$ we ~~$x_2=$~~ would get $1\ 1 \to 1x_1+1x_2=0$
$1(-1)+1(1)=0$.

The eigenvalues and eigenvectors tell us important things about the MATRIX. Notice the matrices $\begin{bmatrix}0&1\\1&0\end{bmatrix}$ and $\begin{bmatrix}3&1\\1&3\end{bmatrix}$. $\begin{bmatrix}3&1\\1&3\end{bmatrix} = \begin{bmatrix}0&1\\1&0\end{bmatrix} + \begin{bmatrix}3&0\\0&3\end{bmatrix}$ or $A-3I=\begin{bmatrix}0&1\\1&0\end{bmatrix}$

The eigenvalues for $\begin{bmatrix}0&1\\1&0\end{bmatrix}$ were 1 and −1.
The eigenvectors were $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}-1\\1\end{bmatrix}$.

For $\begin{bmatrix}3&1\\1&3\end{bmatrix}$, eigenvalues were 4 and 2,
and the eigenvectors were $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}-1\\1\end{bmatrix}$ same?
By adding (shifting) A by 3I, the eigenvalues change by 3. $4=1+3$, $2=-1+3$!

if $A\vec{x}=\lambda\vec{x}$, then $(A+3I)\vec{x} = \lambda\vec{x}+3\vec{x} = (\lambda+3)\vec{x}$



Suppose

suppose you know eigenvalues of B : $\lambda_1=\alpha_1$; $\lambda_2=\alpha_2$
and you know the eigenvalues of A : $\lambda_1, \lambda_2, \ldots$
Do you know eigenvalues of $(A+B)$? NO.

$(A+B)\vec{x} \neq (\lambda+\alpha)\vec{x}$: $\vec{x}$ is not an eigenvector of B.

NOT SO GREAT are $A+B$, AB. For $A+B$ you have to solve the eigenvalue problem ~~det~~ $(A+B)-\lambda I)\vec{x}$ ~~= 0~~ $= 0$ as
$\det((A+B)-\lambda I) = 0$.

example: let Q rotate everything by $90°$.

$$Q = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

trace$(Q) = 0$ = sum of eigenvalues
det(Q) = product of eigenvalues

We know just from these characteristics that the eigenvalues...

trace $\longrightarrow \lambda_1 + \lambda_2$ Something is wrong here?

det $\longrightarrow \lambda_1 \lambda_2$

Eigenvectors come out the same direction they came in.
This cannot happen with the $90°$ rotation.
besides, trace$(Q) = 0 + 0 = 0$
det$(Q) = 1$ Yes. We are in trouble here.

Is there a way out?

$$\det(Q - \lambda I) = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - (-1) = \lambda^2 + 1 = 0$$

holy shit. The eigenvalues are not real numbers.

$\lambda_1 = \hat{i}$ $\qquad (\hat{i})^2 + 1 = -1 + 1 = 0$

$\lambda_2 = -\hat{i}$ $\qquad (-\hat{i})^2 + 1 = -1 + 1 = 0$

$i^2 = -1$

~~doubt~~ it

$i = \sqrt{-1}$

$i = \sqrt{-1}$ is an evil number. I doubt its very existence. It is merely a COMPLEX number.

complex conjugates: a pair with opposite signs

~~It~~ Only in the extreme cases will we have purely imaginary numbers as eigenvalues, like when we are dealing with an antisymmetric matrix ~~set~~ such as the one which generated these two evil mother fuckers.

There is another type that is even worse. Another bad thing that could happen...

Note: There are some great notes in L61 starting on page 102, I may jot down some notes from it to ~~th~~ this coming weekend.

ZONE FIVE

RESTING IN CONFUSION

2000.102.2
04.11.0100

Troubled? Remember what my therapist told me: don't panic. Wait until all grades are in transcript before you hit the panic button. Haven't I already "lost the game"? I am escaping from a life of manual labor, a non-symbol using role...

I will rest in confusion and embrace the complex nature of that which my symbol using brain is being exposed to. I will stubbornly refuse to let some of the "hypersensitive inner turmoil" drive me nuts. If I do no work tonight, so be it. Tomorrow is a full day for t∑ M300.

Tonight I will just read through my notes from Mathematical Reasoning, marking things to be transcribed into t∑.

102. 1100 Resting today. I lay in bed until 11AM. The coffee is on. Mental Health Day. This morning, over and over again in my head were problems from the lalg exam. I wonder why Rutgers "faculty" seems to give off the impression that we the students are lost and confused. We are being processed through a money making machine, albeit a degree granting machine.

Perhaps I did make a mistake in taking all math classes my first semester at Rutgers University. Now I am in it. 6 more lectures left for each course – 3 weeks left and then final exams. My mind certainly has had to stretch these past few months. Non stop, now t5 M300.

Don't worry about the logbooks. They are always changing as I am always changing. I have made much of my daily existence; I write as though one were interested in what I am writing, as though an audience would one day materialize.

How boring to read about ones "things to do" list! And yet this is how I keep my head together, this psychobabble. I am resting in confusion, and about to embrace the complexity of 640:300. As long as I pass these 3 courses, I will be able to really be a free spirit for 7 weeks, preparing for 198:206.

102. 1620 t5 M300 sessions done. I will view Strang lecture on $S^{-1}AS=\Lambda$ until I leave. After class I may go with the flow, trying to juggle all ~~these~~ this knowledge.

Suppose n independent eigenvectors $x_1 \ldots x_n$ put them in columns of S: $AS = A\begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \end{bmatrix} = \begin{bmatrix} \lambda_1 x_1 & \lambda_2 x_2 & \cdots & \lambda_n x_n \end{bmatrix}$

96

$A\vec{x} = \lambda\vec{x}$

$$AS = A\underbrace{\begin{bmatrix} | & & | \\ x_1 & \cdots & x_n \\ | & & | \end{bmatrix}}_{S} = \underbrace{\begin{bmatrix} | & & | \\ \lambda_1 x_1 & \cdots & \lambda_n x_n \\ | & & | \end{bmatrix}}_{AS} = \underbrace{\begin{bmatrix} | & & | \\ x_1 & \cdots & x_2 \\ | & & | \end{bmatrix}}_{S}\underbrace{\begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \lambda_n \end{bmatrix}}_{\Lambda}$$

$AS = S\Lambda$ ⟶ $S^{-1}AS = S^{-1}S\Lambda$

eigenvector MATRIX

capital lambda

$$S^{-1}AS = \Lambda$$

$$A = S\Lambda S^{-1}$$

if $A\vec{x} = \lambda\vec{x}$

$A^2\vec{x} = \lambda \underline{A\vec{x}} = \lambda^2 x$

$A^2 = S\Lambda (S^{-1}S)\Lambda S^{-1} = S\Lambda^2 S^{-1}$

Theorem : $A^k \to 0$ as $k \to \infty$ if all $|\lambda_i| < 1$

A is sure to have n independent eigenvectors and be diagonalizable if all the λ_i are different. (no repeated eigenvalues)

Repeated possibility : if I have repeated eigenvalues, I may or may not have n independent eigenvectors.

A diagonal matrix has its eigenvalues sitting right there in front of you. Not so with a triangular.

Look at $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \longrightarrow$ "trouble"

What are the eigenvalues? $\lambda_1 = 2$, $\lambda_2 = 2$

why? $\det(A - \lambda I) = 0 = \begin{vmatrix} \lambda-2 & 1 \\ 0 & \lambda-2 \end{vmatrix} = (\lambda-2)^2 = (\lambda-2)(\lambda-2)$

the eigenvectors

$A - 2I = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ The algebraic multiplicity is 2

$(\lambda-2)$ is a "double root"

What's in the nullspace? $x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is it.

103.2310 Brain overloading, but do not count me out. I have so much to go over for Mathematical Reasoning, but I am amazed with other "pre-calculus" notions coming to the surface as keys to solving triple integrals:

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

MAJOR IDENTITIES (besides $\sin^2\theta + \cos^2\theta = 1$)

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \sin\phi\cos\theta$$
$$\cos(\theta+\phi) = \cancel{\sin}\ \cos\theta\cos\phi - \sin\phi\ \cancel{\cos}\ \sin\theta$$

$$\tan(\theta+\phi) = \frac{\sin(\theta+\phi)}{\cos(\theta+\phi)} = \frac{\sin\theta\cos\phi+\sin\phi\cos\theta}{\cos\theta\cos\phi-\sin\theta\sin\phi}$$

divide numerator and denominator by $\cos\theta\cos\phi$

$$\tan(\theta+\phi) = \frac{\frac{\sin\theta}{\cos\theta}+\frac{\sin\phi}{\cos\phi}}{1-\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\sin\phi}{\cos\phi}\right)} = \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$$

The subtraction formulas follow from the addition.

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

observe:
$$\cos^2\theta + \sin^2\theta = 1$$
$$2\sin^2\theta = 1-\cos(2\theta)$$
$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$
$$2\cos^2\theta = 1+\cos(2\theta)$$

glimpse p100. That awaits me, but I want to make sense of Trigonometric Identities so as to not get tripped up.

Think basic: $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta - \sin^2\theta = \cos(2\theta)$

If I rewrite these, when I add and subtract, I get the very useful information I will need to calculate $\int \sin^2\theta \, d\theta$ and $\int \cos^2\theta \, d\theta$

$$\begin{aligned} &\cos^2\theta + \sin^2\theta = 1 \\ + \; &\cos^2\theta - \sin^2\theta = \cos(2\theta) \\ \hline &2\cos^2\theta = 1 + \cos(2\theta) \end{aligned}$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

$$\begin{aligned} &\cos^2\theta + \sin^2\theta = 1 \\ - \; &\cos^2\theta - \sin^2\theta = \cos(2\theta) \\ \hline &2\sin^2\theta = 1 - \cos(2\theta) \end{aligned}$$

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\therefore \int \cos^2\theta \, d\theta = \int \frac{1}{2} + \frac{\cos(2\theta)}{2} \, d\theta = \frac{\theta}{2} + \frac{\sin(2\theta)}{4}$$

this is the same as $\dfrac{\sin(\theta)\cos(\theta)}{2} + \dfrac{\theta}{2}$

so $\dfrac{\sin(2\theta)}{4} = \dfrac{\sin(\theta)\cos(\theta)}{2}$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ aha...

This is TRIGONOMETRY ... still, forever a Primary Role

Ofcourse, these make differentiating easier as well.

$$\frac{d}{d\theta}\left(\cos^2\theta\right) = \frac{d}{d\theta}\left(\frac{1}{2} + \frac{\cos(2\theta)}{2}\right) = 0 + \left(\frac{-\sin(2\theta) * 2}{2}\right)$$

$$= -\sin(2\theta)$$

which is also $-2\sin\theta\cos\theta$

The other simple identity derived from subtracting $\cos(2\theta)$ from 1 is just as useful.

$$\int \sin^2\theta \, d\theta = \int \frac{1}{2} - \frac{\cos(2\theta)}{2} \, d\theta = \frac{\theta}{2} + \frac{\sin(2\theta)}{4}$$

which equals $\frac{\theta}{2} + \frac{\sin\theta\cos\theta}{2}$

and $\frac{d}{d\theta}\left(\sin^2\theta\right) = \frac{d}{d\theta}\left(\frac{1}{2} - \frac{\cos(2\theta)}{2}\right) = 0 + \frac{\sin(2\theta) * 2}{2}$

$$= \sin(2\theta) = 2\sin\theta\cos\theta$$

Trivial? perhaps, but crucial. Now, it is almost midnight. I will be up for at least another couple hours going over t's for yet another Math class exam. To think, my nephew is just beginning his journey. I told him that his study of Calculus begins with his in depth review of Algebra, Trigonometry, and Pre-Calculus functions. I believe he will really get into this NON-religious technical/mathematical education. In my head is $f(A \cap B) = f(A) \cap f(B)$ iff f is 1-1.

$(A \cap B) \subseteq A$, $(A \cap B) \subseteq B$, $f(A \cap B) \subseteq f(A)$, $f(A \cap B) \subseteq f(B)$

Suppose $f(x_1) = f(x_2) \rightarrow x_1 = x_2$. Then $y \in (f(A) \cap f(B))$

$y \in f(A)$ $y \in f(B)$ $y = f(a)$ for $a \in A$ $y = f(b)$ for $b \in B$

$y = f(a) = f(b)$ ∴ $a = b \in (A \cap B)$ $y = f(a)$ for $a \in A$ and $a \in B$

since f is injective hence $f(A \cap B) = f(A) \cap f(B)$

104

But there is more: the contrapositive.
Suppose $f(A \cap B) = f(A) \cap f(B)$ for any $A, B \subseteq X$
suppose $f(x) = f(x')$ but $x \neq x'$
Take $A = \{x\}$ $B = \{x'\}$ $A \cap B = \emptyset$
then $f(A) = \{f(x)\}$ $f(B) = \{f(x')\}$
now, $f(x) = f(x')$ so $f(A) = f(B)$
and $f(A) \cap f(B) = \{f(x)\}$
but $f(A \cap B) = 0$
so $f(A \cap B) \subseteq f(A) \cap f(B)$ if f not injective.

I will relax and keep my wits about me. Even if I get bad news tomorrow morning with Linear Algebra results (exams). I have another exam tomorrow to get through, and then I can devote my brain to multivariable calculus for the next few days.

2000.104.4
04.13.0030

I really hope everything works out for my nephew. I know for sure that he ~~will~~ would enjoy Elementary Algebra more than Calculus. I would like to see him get bumped into Intermediate Algebra via exam, but whatever happens, happens. He will be so much more prepared for Calculus with the strong foundation math 151, 152, 153. This will stretch his years at BCC, but, so be it. I am proud of him. He is embracing this obstacle/crisis as an opportunity to build solid skills

Anyway, I will jot down some thoughts off the cuff. If $m \leq_2 n$ and $n \leq_2 m$
let $m = na$ let $n = bm$ (my mnemonic device (shit)
$na \rightarrow$ dope $\rightarrow$ shit $\rightarrow bm$)
then $m = abm$ $ab = 1$
or $m = 0 = n$. else $a = b = 1$ so $m = n$
if $m \leq_2 n$ and $n \leq_2 p$ $n = ma$ $p = nb$ $p = mab$ $\therefore m \leq_2 p$

104.1330 No exam results for M250, but the lecture was brutal. I will not be able to put off 6.2. I will hold off on 6.3 until Wednesday night, after Calculus exam#2. The schedule on page 101 is accurate for the moment. I will follow it.

104.1500 I got caught up on the phone with my nephew. He did well on the elementary algebra placement exam. He does not have to take math 025! He does not even want to take ~~mid~~ the advanced placement exam for ~~a~~ intermediate algebra, as he is afraid he might get lucky and pass it. He WANTS to learn. This is great.

Now, I have a couple hours before I take the M300 exam. I am a little nervous.

104.1630 I am very happy for my nephew. He had been against the idea of taking Pre-Calculus before, but now, because he is not required to take elementary algebra, is enthusiastic about taking Intermediate Algebra. He will get the book in Advance and study it over the summer.

Now, I put in a call to University College — they will see about processing my application for a dean to dean transfer. They are impressed with my gpa from Brookdale... 3.8.

Well, I plan to head over to Dunkin Donuts early, at about 520... I will sit down and eat some donuts with my coffee while going over my M300 tS notes.

After the exam, I will pick up some take out food at the Commons, and then I will be ready to settle down and start the Calc3 X2 tSessions.

Now, what about $x \in X$ $[x] = \{y \in X \mid (x,y) \in$

Well, each $[x] \neq 0$ since for each $x \in [x]$, $(x,x) \in X$

Each $x \in X$ is in some equivalence class.

for $x, y \in X$ either $[x] = [y]$ or $[x] \wedge [y] = 0$

Suppose that $[x] \wedge [y] \neq 0$ that $z \in [x] \wedge [y]$

then $(x,z) \in X$ and $(y,z) \in X$

so $(z,x) \in X$ and $(z,y) \in X$ by symmetry

then $(x,y) \in X$ by transitivity.

Actually $[x] = [y]$ if $(x,y) \in X$, but we can make it a stronger proof by supposing $(x,t) \in X$

then $(t,x) \in X$ by symmetry $t \in [x]$

and $(\cancel{x}, t) \in X$ by transitivity.

so $t \in [y]$

hence $[x] \subseteq [y]$

$[y] \subseteq [x]$ and $[x] = [y]$

104. 2230 The M300 exam was ok. I predict another 80 or 85. I finished one calc3 problem in TS, 1 of 15. Just a note here:

$D_{\vec{u}_1}(D_{\vec{u}_2} f)$ involves

$$\hat{u}_1 = \frac{\vec{u}_1}{\|\vec{u}_1\|} \qquad \hat{u}_2 = \frac{\vec{u}_2}{\|\vec{u}_2\|} \qquad \vec{\nabla} f = \langle f_x, f_y \rangle$$

$$D_{\vec{u}_2} f = \hat{u}_2 \cdot \vec{\nabla} f$$

Now $D_{\vec{u}_1}(D_{\vec{u}_2} f)$ requires $\vec{\nabla}(D_{\vec{u}_2} f)$

and then $\hat{u}_1 \cdot \vec{\nabla}(D_{\vec{u}_2} f)$

If I keep my head together, this type of problem will be purely computational and the MECHANICS of solving it will be a harmonious algorithm that will flow gracefully...

105.1230 $\$ \dot{C}$ balance 350. Bills to pay phone 90, surcharge 30.

$$\begin{array}{r} 350 \\ -120 \\ \hline 230 \end{array}$$

to deposit 300 DVR check → 530

Note: Do not use calling card – Too expensive.
Get bill down to \$50.00.

I will mail out these 2 checks this week.

105.1630 The quadratic formula is derived by completing the square.

notice $(x+A)^2 = x^2 + 2Ax + A^2$

The constant is "$\frac{1}{2}$ the coefficient of x" squared.

This single observation is the basis for completing the square.

We derive the quadratic formula like so:

$$ax^2 + bx + c = 0 = x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$\frac{1}{2}$ the coefficient of x squared $\rightarrow \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

move constant to left side of equation

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

complete the square and add it to both sides:

$$\underbrace{x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}}_{\text{perfect square}} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$\frac{1}{2}$ coefficient of x, $\frac{b}{a}$

Take the square root of both sides: $x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{\pm\sqrt{b^2-4ac}}{2a} - \frac{b}{2a} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

! NO LONGER A MYSTERY

I really do love math. I guess I have just been a little overwhelmed these past few months. Even though I no longer plan to minor in mathematics, and although the definition of MATHEMATICIAN is "an expert or specialist in mathematics", as a computer scientist or a scientist in general, mathematics is at the core of my "specialty", "trade", CRAFT.

This latest insight into the ROOTS of the quadratic formula, along with the useful method of deriving the identities of $\sin^2\theta$ and $\cos^2\theta$ as shown on pages 102 to 104 (L62 * this), truly inspire me.

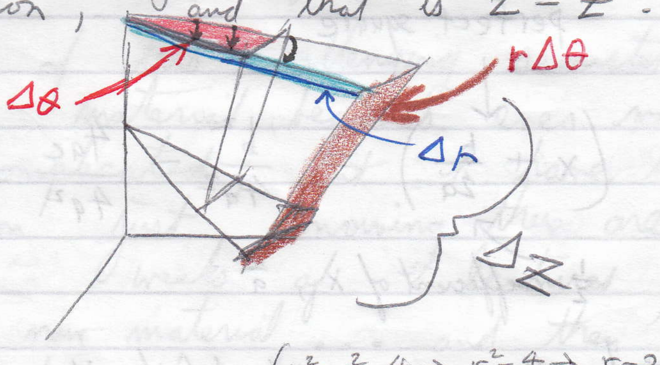
I feel like I won't have time to get to LAlg 6.2 tonight. Should I view Strang Lecture while I am working on tSC3X2?

2000.106.6
04.15.0030

I finished 13 of the 15 problems in tSC3X2. The last 2 involve cylindrical and spherical coordinates. Shall I work out some fundamental ideas about this here in L62 before attempting sessions on them in tS? Yes. See p.100 L62.

Cylindrical coordinates is just polar coordinates with one more dimension, and that is $Z = Z$.

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

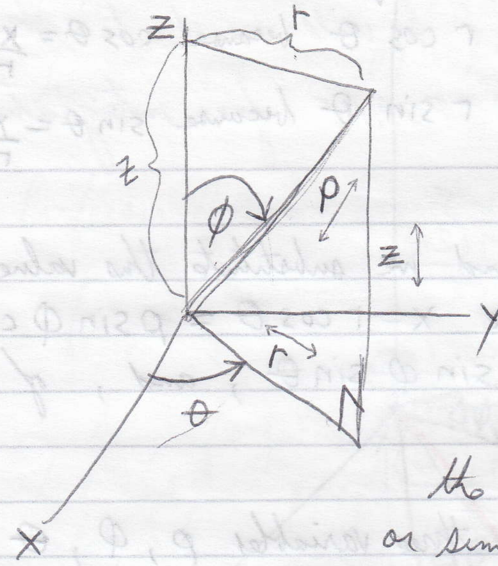


$f(x,y,z) \longrightarrow f(r,\theta,z)$

$(x^2+y^2=4 \Rightarrow r^2=4 \Rightarrow r=2)$

$$\int_{y=0}^{2}\int_{x=0}^{\sqrt{4-y^2}}\int_{0}^{4+x^2+y^2} dz\,dx\,dy = \int_{\theta=0}^{\pi/2}\int_{r=0}^{2}\int_{z=0}^{4+r^2} r\,dz\,dr\,d\theta$$

Spherical coordinates



Given $f(\rho, \phi, \theta)$

$= f(\text{rho}, \text{phi}, \text{theta})$

$= f(\rho, \varphi, \theta)$

rho, ρ, is the distance from the origin $(0,0,0)$ to the point (x, y, z), or simply $\sqrt{x^2+y^2+z^2} = \text{rho} = \rho$

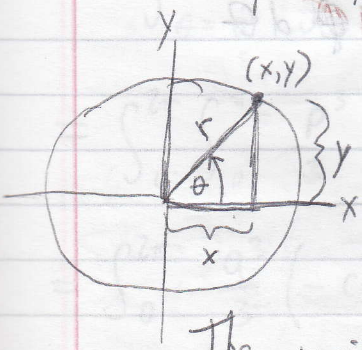
θ is an old friend, the same as in polar coordinates, positive in a counterclockwise direction from zero to 2π. (~~2PA~~ $2\pi \to$ like 2PAC, what a pop name!)

φ, phi – pronounced like Fili without the li.
Fi $\to$ fee $\to$ phi $\to$ $\phi = \varphi$.
φ is the angle measured in a positive sense from the positive z-axis down and goes from zero to π.

grok this : $z = \rho \cos \varphi$

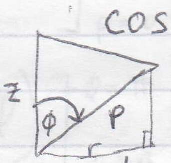
let us reflect upon $x = r\cos\theta$, and perhaps we shall FEEL THIS.

$\cos\theta = \dfrac{x}{r} \Rightarrow r\cos\theta = x$



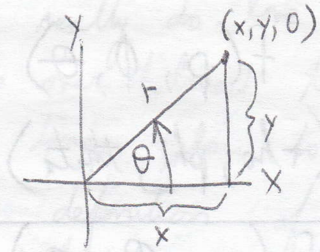
so, what about $z = \rho \cos \varphi$?

$\cos \varphi = \dfrac{z}{\rho}$ $\leftarrow$ the "adjacent ~~angle~~ side" / $\leftarrow$ the "hypotenuse"



The projection of ρ into the x-y plane, r, like in polar coordinates, is $r = \rho \sin \varphi$

$\sin \varphi = \dfrac{r}{\rho}$ $\leftarrow$ "opposite φ" / $\leftarrow$ "hyp"



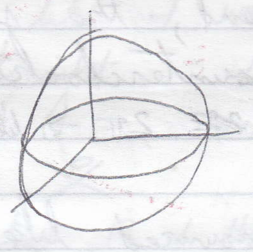
flat, looking down.

$x = r\cos\theta$ because $\cos\theta = \frac{x}{r}$

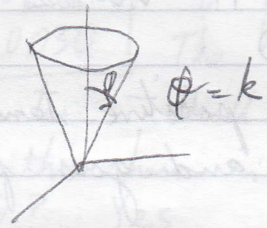
$y = r\sin\theta$ because $\sin\theta = \frac{y}{r}$

Now, $r = \rho\sin\phi$, and we substitute this value in the above to derive $x = r\cos\theta = \rho\sin\phi\cos\theta$ and $y = r\sin\theta = \rho\sin\phi\sin\theta$, and, of course, $z = \rho\cos\phi$.

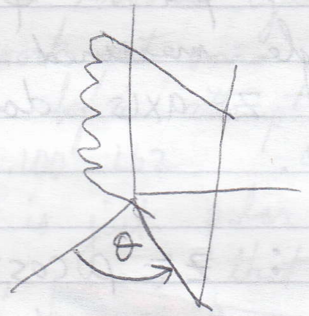
Here is what each of the three variables ρ, ϕ, θ look like as constants:



$\rho = k$



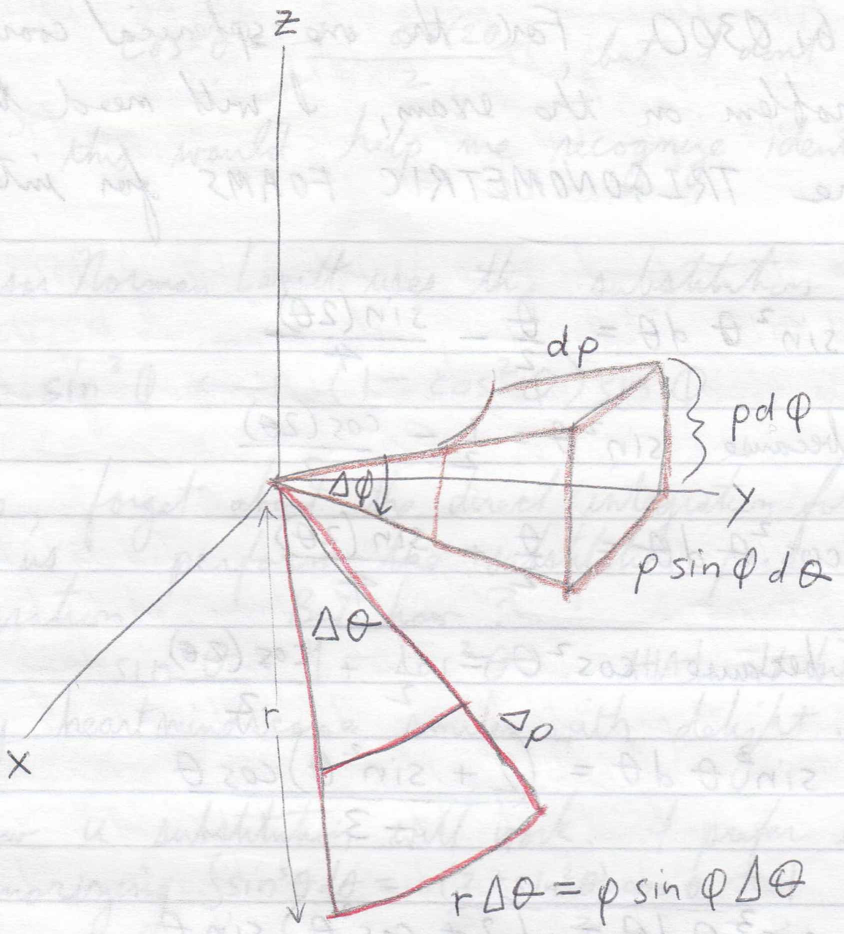
$\phi = k$



$\theta = k$

$$dz\,dy\,dx \longrightarrow \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

break it down?



example: find volume of sphere with radius a
We know the answer is $V = \left(\frac{4}{3}\pi a^3\right)$

$$\int_{x=0}^{x=a}\int_{y=0}^{y=\sqrt{a^2-x^2}}\int_{z=0}^{z=\sqrt{a^2-x^2-y^2}} 1 \, dz\, dy\, dx$$

radius of sphere

$$= \int_{\theta=0}^{\theta=2\pi}\int_{\varphi=0}^{\varphi=\pi}\int_{\rho=0}^{\rho=a} \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\rho^3}{3}\sin\varphi \Big]_{\rho=0}^{\rho=a} d\varphi\, d\theta = \int_0^{2\pi}\int_0^{\pi} \frac{a^3}{3}\sin\varphi \, d\varphi\, d\theta$$

$$= \int_0^{2\pi} \frac{a^3}{3}\left(-\cos\varphi\right)\Big]_{\varphi=0}^{\varphi=\pi} d\theta = \frac{a^3}{3}\int_0^{2\pi}\left(-\cos\pi + \cos 0\right) d\theta$$

$$= \frac{a^3}{3}\int_0^{2\pi}\left(-(-1)+1\right) d\theta = \frac{a^3}{3}\int_0^{2\pi} 2\, d\theta = \frac{2a^3}{3}(\theta)\Big[_{\theta=0}^{\theta=2\pi} = \frac{4\pi a^3}{3}$$

#11

106.0300 For the one spherical coordinates problem on the exam, I will need to know some TRIGONOMETRIC FORMS for integration

① $\int \sin^2\theta \, d\theta = \frac{\theta}{2} - \frac{\sin(2\theta)}{4}$

because $\sin^2\theta = \frac{1}{2} - \frac{\cos(2\theta)}{2}$

② $\int \cos^2\theta \, d\theta = \frac{\theta}{2} + \frac{\sin(2\theta)}{4}$

because $\cos^2\theta = \frac{1}{2} + \frac{\cos(2\theta)}{2}$

③ $\int \sin^3\theta \, d\theta = \frac{(2+\sin^2\theta)\cos\theta}{-3}$

④ $\int \cos^3\theta \, d\theta = \frac{(2+\cos^2\theta)\sin\theta}{3}$

We use the fundamental identity $\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

so, $\cos^3\theta = (\cos^2\theta)(\cos\theta) = (1-\sin^2\theta)\cos\theta$

if we let $u = \sin\theta$, then $du = \cos\theta$

$\int (1-u^2)\,du = u - \frac{u^3}{3} = \sin\theta - \frac{\sin^3\theta}{3}$

so why does $\frac{(2+\cos^2\theta)\sin\theta}{3} = \sin\theta - \frac{\sin^3\theta}{3}$?

well, $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$, but I don't see how this would help me recognize identity!

Professor Norman Levitt uses this substitution for $\sin^3 \theta \longleftrightarrow (1-\cos^2\theta)\sin\theta$

So, forget about the direct integration formula. Let us perform the substitution prior to the integration. But how?

$\sin^2\theta = 1-\cos^2\theta$ AHA! It bit my NOSE!

My heartmindscape smiles with delight.

Now u substitution will work. I prefer it to "memorizeing $\int \sin^3\theta \, d\theta = \frac{(2+\sin^2\theta)\cos\theta}{3}$!"

$\int (1-\cos^2\theta)\sin\theta$ let $u=\cos\theta$ then $du = -\sin\theta$

$-\int 1-u^2 \, du = -\left(u - \frac{u^3}{3}\right)$

$= \boxed{\frac{\cos^3\theta}{3} - \cos\theta}$ Beautiful!

p56 Integration By Parts $\longrightarrow$ t5#044 Review

106.0415 ... soon to fall asleep, I finished t5C3X2. Now, tomorrow afternoon, when I awaken, wonder, and despair, I can put on some coffee and start chewing on some Linear Algebra section 6.2 as well as starting Lab #6 (LAST ONE).
I will also look over t5C3X2.

NERVE NET

107, 1300 B&N

$(a+b)^0 = 1$

$(a+b)^1 = 1a + 1b$

$(a+b)^2 = 1a^2 + 2ab + 1b^2$

$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{n-1} a b^{n-1} + \binom{n}{n} b^n$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$ whichever $a^n k^{n-k}$ works just as well

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1} \longrightarrow \underset{10}{\binom{5}{3}} + \underset{5}{\binom{5}{4}} = \binom{6}{4} \quad 15$$

$\frac{6!}{4!}$

$$\binom{6}{4} = \frac{6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2}}{(\cancel{4 \cdot 3 \cdot 2})(2)} = 15 \qquad \frac{5 \cdot 4 \cdot 3 \cdot 2}{(4 \cdot 3 \cdot 2)(1)} = 5 \qquad \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{4 \cdot 4 \cdot 3 \cdot 2 \cdot 2}$$

No proofs available. I may work on this Tuesday...

Tangent: ROOTS   I will use one I know:

$$\begin{array}{r} 16 \\ \times 16 \\ \hline 96 \\ 160 \\ \hline \boxed{256} \end{array} \qquad \begin{array}{r} 256 \\ \times 256 \\ \hline 1536 \\ 12800 \\ 51200 \\ \hline 65536 \end{array}$$

$\sqrt{65536}$

Step 1: divide Radicand into groups of 2 digits.

(extraction galley) use prime (') to separate digits

$$\begin{array}{r} 8 \\ 8 \\ \hline 16 \end{array}$$

$8^2 = 64$

$\sqrt{66'53'6.6}$
64
new minuend → 253

Step 2: Find largest number whose square is less than the leading two digit group (64)
I will do the work and explain later.
$8^2 = 64 < 66$

set up from the decimal point!

$\sqrt{6'55'36.}$

extraction galley

2
+ 2
45
+ 5
506
6

$\sqrt{06'55'36.} = 256$

$2*2=4$

~~4~~
~~2~~55
225

$45*5 = 225$

~~2~~3036
36

$2^2 = 4 < 6$

$506*6 = 3036$

one more time: $\sqrt{65536}$

+2
2
45
5
506
6

$2*2=4$

$\sqrt{6'55'36} = 256$
4
255
$45*5=225$
225
3036

What number, when ~~added to multiplied~~ added to 40 ?

$506*6=3036$

1
+ 1
24
4
281
1
2824

$\sqrt{2.'00'00} = 1.414$

$1*1 = 1$
100
$24*4$
96
400
$281*1$
281
11900
$2824*4$
11296
604

8000
3200
80
16
11296

what number times 280 is less than 400 ? 1

What number times 2820 is less than 11,900 ? 4

This was just a tangent. It is useful for simple calculat

1
+1
26
326
6
156

$\sqrt{2'56} = 1$
1
156

What number times 20 is less than 156 ? plus this number.

$\boxed{\begin{matrix}2000.109.2\\04.18.0200\end{matrix}}$ I am calling it a night. I will read some science fiction as I drift to sleep. Before I complete the next 5 X2 problems, I will try to get a grip on Linear Algebra 6.3 "Applications to Differential Equations", especially the exponentials of a matrix. I will study the chapter and attempt the excercises.

I still want to go over more Calculus 3, but enough is enough for tonight.

I do want to make a note on how to handle $\int \cos^4\theta \, d\theta$.

$$\int_0^{2\pi}\int_0^{\cos^2\theta} r\,dr\,d\theta = \int_0^{2\pi} \frac{r^2}{2}\Big[_{r=0}^{r=\cos^2\theta} \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \cos^4\theta \, d\theta = \frac{1}{2}\int_0^{2\pi}\left(\frac{1}{2} + \frac{\cos(2\theta)}{2}\right)^2 d\theta$$

see p 103 L62

$$= \frac{1}{2}\int_0^{2\pi} \frac{1}{4}\left(1+\cos(2\theta)\right)^2 d\theta$$

$$= \frac{1}{8}\int_0^{2\pi} 1 + 2\cos(2\theta) + \cos^2(2\theta) \, d\theta$$

$$= \frac{1}{8}\int_0^{2\pi} 1 + 2\cos(2\theta) + \frac{1}{2} + \frac{\cos(4\theta)}{2} \, d\theta$$

$$= \frac{1}{8}\int_0^{2\pi} \frac{3}{2} + 2\cos(2\theta) + \frac{\cos(4\theta)}{2} \, d\theta$$

$$= \frac{1}{8}\left[\frac{3\theta}{2} + \frac{2}{2}\sin(2\theta) + \frac{\sin(4\theta)}{8}\right]_{\theta=0}^{\theta=2\pi}$$

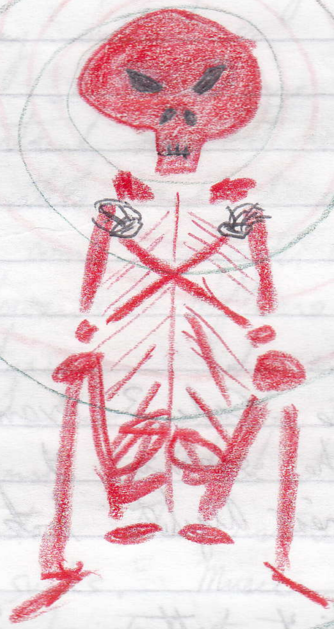
$\sin 0 = 0$
$\sin 2\pi n = 0$

$$= \frac{1}{8}\left(3\pi + 0 + 0\right) = \frac{3\pi}{8}$$

zone seven

LIFE SUCKS

I beg to differ!
MWH 2000.140
A 640:251
B+ 640:300
B+ 640:250



yes, it certainly does suck for everyone sometimes
no one escapes the SUCKINESS provided to us by our frustrated desires . . .

desire causes suffering

Discard expectations... they lead directly to your own personal Hell.

"So Farewell hope!
And with hope, farewell to fear
farewell remorse!
Evil, be thou my good." —satan

This is the cry of outraged innocence — albert camus

Failure teaches me to be happy with just getting by.
Ambition is ugly. It eats a man alive from the inside out. Blessed be the slacker!

117. 2340 The last sheet of papyrus of any volume of my memoirs deserves to be devoted to some sort of broader scoped vision. Yes, I am currently devoured by my academic reality, the pressure of final exams; but isn't there a "larger picture"?

The way my sister and I embraced today, after not speaking to one another since before this semester began (back during the second week of January!!), exactly on the last day of my memoirs in L_{62}, is eerie; it is like some surrealistic ending of a chapter...

And what a new chapter on the horizon --- first final exams at Rutgers over the next two weeks, graduation at Brookdale ~~Communication~~ Community College a week after that, and I ease into L_{64}... L_{63} promises to be ~~a~~ a packed "chapter"/logbook. How far I have come since the summer of 1997, since the summer of 1998 even!

No matter what happens over the next two weeks, during "the first half of EMBRACING COMPLEXITY", I want to remember how far I have come. I want to be fully aware of the odds I have triumphed against. Surely, this journey towards ~~metamorphic~~ becoming a professional scientist, from a lowly crack-head/janitor has not been quick nor swift. In fact, I started the adventure back in 1993 while all was well with Mikey, Sherry, Ginger, Sparkle, and Forest in the Tack House @ Monmouth Battlefield State Park. Was it Logbook #39 or 40? Scribblings? I became very interested in mathematics again. I purchased a computer. In 1994 I signed up for Calculus I at the local community college... then Calculus II... then I kicked Sherry out and got drunk-

172

In 1995 I switched my majors from Mathematics to Computer Science based on the advice from Calculus instructor Jay Dashabundu. He said that computer science was more marketable than mathematics alone, and that at the heart of computer science was pure mathematics.

And so, though drunk, stoned, and often skied, I made my way through Introduction to Digital Programming, Computer Concepts, and Pascal. By then, 1996, Mary Moss had conned her way into the Tark House, and I was at my WITS end, fully psychotic via crack cocaine and alcohol and constant TCP smoke. I could no longer afford to go to Brookdale as I had too many expensive habits. I went bankrupt as I kept going from the ATM (with my credit card) to the street... the forbidden Black Street, to be served on a regular basis.

Suicidal in 1996 and diagonosed manic-depressive, it would get worse before it got better. I have a history. I am no typical college student. I am a truly NON-TRADITIONAL STUDENT.

By July 1997 I was incarserated for the second time in my life (first time in 1987 for 18 months). This time 7 months for "eluding the police", "wreckless driving", "resisting arrest". By the end of the 7 months I had lost my job of 8 years with the State Park Service. I lost the Tark House and my drivers liceanse. By the summer of 1998 I was a full time computer science ~~st~~ student, and I have been ever since.

PREV L61 2000.041..081

Useful Information

NEXT L63

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60

CONVERSION TABLES

LENGTH

1 meter (m) = 100 cm = 1,000 mm
1 millimeter (mm) = .001 m
1 centimeter (cm) = .01 m
1 decimeter (dm) = .1 m

Dear Tami,

This is spontaneous, so I will try to be brief, honest, and clear.
I will be careful with my words, as "I love you" seems as commonplace as "hello".

I am pained by the present distance between us.
I want you to know that I am not an unfeeling monster.
I also realize that neither you nor your husband Joe are unfeeling monsters either. This present state of exile from you is painful, and I am not so cold-hearted as to not be bruised by Joe's hatefulness towards me.

I am not writing to you out of self-pity. I am just writing without committing myself to actually hitting the send button. I do intend on sending this message out.

I will not defend myself or ask to be forgiven, nor do I wish to throw stones at you. Please receive my words as simple noises made. How I envy the whales, that they can communicate such emotion. I am only a man. Words are insufficient.

The world does not center around me, as you well know; but I shall go on living, caught in the web of my individual existence. You especially, but your husband Joe as well, in the fabric of my memories, are very much part of my inner reality. This will never change, no matter what.

In a tender moment, I am sending out the closest thing to a whale's cry that I can.

This message may not change anything on an external level, and life will go on and on as usual - but as time goes on, some memories weaken, while some memories become stronger. How I am represented in your memories is none of my business. Fortunately, nothing can change the fact that we are brother and sister. I do not want to accept the present disharmony, but what choice do I have? I have to go on living.
As important as you and your husband are to me, I had better not put too much importance on your opinions of me. Your opinions of me do matter to me, but - as they are yours, they belong to you. I have no desire to alter your honest perception of me, nor do I apologize for my individuality. I accept the consequences of my flaws, quirks, weaknesses, and strengths.

How I have complicated a simple, tender moment!
This is the surface of my feelings. I promised to be brief.
I better try to get some sleep.
Your brother,
mike

13½ cu. ft. of air weighs 1 lb.

A township is 36 sq. miles.

3.785	8.000	4.000	1.000